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Corrigendum

Analysis of fluid equations by group methods

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Sections 2–4 should be replaced by the text below.

2. Burger's equation

For the Burgers' equation

$$u_t + uu_x = -p_x + \mu u_{xx} \tag{2.1}$$

the full two-parameter group (α, β) with two arbitrary functions, $f(t)$ and $j(t)$, is

$$\begin{aligned} T &= \alpha + 2\beta t, \quad X = \beta x + f(t), \\ U &= -\beta u + f'(t), \quad P = -2\beta p + j(t) - xf''. \end{aligned} \tag{2.2}$$

With $\alpha = 1$, $\beta = 0$ the subgroup

$$T = 1, \quad X = f(t), \quad U = f'(t), \quad p = j(t) - xf''$$

has the generator

$$QI = \frac{\partial I}{\partial t} + f(t)\frac{\partial I}{\partial x} + f'(t)\frac{\partial I}{\partial u} + [j(t) - xf'']\frac{\partial I}{\partial p} = 0. \tag{2.3}$$

The Lagrange equations of (2.3) yield

$$\bar{u} = u - f(t), \quad \bar{x} = x - F(t), \quad \bar{p} = p + xf'(t) - k(t), \tag{2.4}$$

where $F' = f$ and $k(t) = \frac{1}{2}f^2 + \int j(t)\mathrm{d}t$. Applying (2.4) to (2.1) results in

$$\bar{u}\bar{u}_x = -\bar{p}_{\bar{x}} + \mu\bar{u}_{\bar{x}\bar{x}}, \tag{2.5}$$

that is, the steady equation. One integration gives the Riccati equation

$$U'(\bar{x}) + U^2 = \lambda\bar{p}(\bar{x}) + c, \quad \lambda = 2/(4\mu)^2, \tag{2.6}$$

where $\bar{u} = -2\mu U$ and c is constant.

3. The Korteweg-de Vries equation

Under the transformation (2.4) the KdV equation

$$u_t + uu_x = u_{xxx} - p_x$$

becomes

$$\bar{u}\bar{u}_{\bar{x}} = \bar{u}_{\bar{x}\bar{x}\bar{x}} - \bar{p}_{\bar{x}}$$

which has the first integral

$$\tfrac{1}{2}\bar{u}^2 = \bar{u}_{\bar{x}\bar{x}} - \bar{p} + c.$$

4. The equation $u_t + uu_x = [\phi(u_x)u_x]_x - p_x$

The action of (2.4) transforms the equation of the title into

$$\bar{u}\bar{u}_x = [\phi(\bar{u}_{\bar{x}})\bar{u}_{\bar{x}}]_{\bar{x}} - \bar{p}_{\bar{x}}.$$